Benjamin Walter **math210**
Assignment Fast_Fourier due 02/02/2022 at 02:13pm EET

Problem 1. **(1 point)** METUNCC/Applied_Math/fourier/Fast_Fourier.pg

In this problem you will apply the fast Fourier transform to

$\vec{\mathbf{f}} = \langle 3,\ 3,\ -2,\ 3,\ 1,\ -1,\ -2,\ 1 \rangle$

(You must write your answers without complex multiplication, trig functions, or powers.)

(A) Split $\vec{\mathbf{f}}$ into its even and odd components:

$\vec{\mathbf{f}}_{\text{even}} = \langle \_\_,\ \_\_,\ \_\_,\ \_\_ \rangle$
$\vec{\mathbf{f}}_{\text{odd}} = \langle \_\_,\ \_\_,\ \_\_,\ \_\_ \rangle$

(B) Compute the Fourier transforms of the even and odd components:

$\mathcal{F}\left\{\vec{\mathbf{f}}_{\text{even}}\right\} = \langle \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_ \rangle$
$\mathcal{F}\left\{\vec{\mathbf{f}}_{\text{odd}}\right\} = \langle \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_ \rangle$

(C) Combine the Fourier transforms of the even and odd components to get the transform of $\vec{\mathbf{f}}$

$\mathcal{F}_0\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ + \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_1\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ + \overline{\omega}\ \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_2\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ + \overline{\omega}^2\ \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_3\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ + \overline{\omega}^3\ \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_4\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ - \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_5\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ - \overline{\omega}\ \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_6\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ - \overline{\omega}^2\ \_\_\_\_ \right) = \_\_\_\_\_$
$\mathcal{F}_7\left\{\vec{\mathbf{f}}\right\} = \frac{1}{2}\left(\_\_\_\_ - \overline{\omega}^3\ \_\_\_\_ \right) = \_\_\_\_\_$

Problem 2. **(1 point)** METUNCC/Applied_Math/fourier/Fast_Fourier_inv.pg

In this problem you will apply the inverse fast Fourier transform to

$\vec{\mathbf{c}} = \big(2,\ -1-2i,\ 1,\ -3-2i,\ 0,\ -3+2i,\ 1,\ -1+2i\big)$

(A) Split $\vec{\mathbf{c}}$ into its even and odd components:

$\vec{\mathbf{c}}_{\text{even}} = \big(\ \_\_,\ \_\_,\ \_\_,\ \_\_\big)$

$\vec{\mathbf{c}}_{\text{odd}} = \big(\ \_\_,\ \_\_,\ \_\_,\ \_\_\big)$

(B) Compute the inverse Fourier transforms of the even and odd components:

$\mathcal{F}^{-1}\left\{\vec{\mathbf{c}}_{\text{even}}\right\} = \big(\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_\big)$

$\mathcal{F}^{-1}\left\{\vec{\mathbf{c}}_{\text{odd}}\right\} = \big(\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_\big)$

(C) Combine the inverse Fourier transforms of the even and odd components to get the transform of $\vec{\mathbf{c}}$

$\mathcal{F}_0^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ + \quad \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_1^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ + \ \omega\ \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_2^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ + \ \omega^2\ \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_3^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ + \ \omega^3\ \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_4^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ - \quad \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_5^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ - \ \omega\ \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_6^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ - \ \omega^2\ \_\_\_\_ = \_\_\_\_\_$

$\mathcal{F}_7^{-1}\left\{\vec{\mathbf{c}}\right\} = \_\_\_\_ - \ \omega^3\ \_\_\_\_ = \_\_\_\_\_$

Problem 3. **(1 point)** METUNCC/Applied_Math/fourier/Fast_Coeff.pg

Suppose that $\mathbf{f}$ is a vector of length 12 with $\mathcal{F}_1\{\mathbf{f}_{\mathbf{even}}\} = 2-2i$ and $\mathcal{F}_1\{\mathbf{f}_{\mathbf{odd}}\} = -3-i$.

Compute the following.

- $\mathcal{F}_1\{\mathbf{f}\} =$ _____
- $\mathcal{F}_5\{\mathbf{f}\} =$ _____
- $\mathcal{F}_7\{\mathbf{f}\} =$ _____
- $\mathcal{F}_{11}\{\mathbf{f}\} =$ _____

(Your answer cannot use complex multiplication, powers, or trig functions.)